

Final Exam

1. Macroeconomic Accounting (15 points)

Consider the following data for our idealized economy with three goods: gismos, widgets, and doodads.

	2015	
	Quantities	Prices
Gismos	4	\$2
Widgets	1	\$5
Doodads	8	\$6

	2016	
	Quantities	Prices
Gismos	5	\$4
Widgets	3	\$7
Doodads	4	\$2

- (a) Using the chain weighting method with 2015 as the base year, compute the real GDP in 2015 and 2016.
- (b) What is the real GDP growth rate (in %) from 2015 to 2016 (use the chain weighting method exactly as in part a). Compare this value to the nominal GDP growth rate (in %) from 2015 to 2016.

2. Endogenous Growth Theory (35 points)

Can the government do better than the private sector?

Consider the growth model with technological change from class. A government is added to the model that taxes firm profit at the rate τ and uses the tax revenue to increase the growth of varieties according to the following equation:

$$N(t+1) = \left(1 + \eta L_R(t) + \phi \frac{g(t)}{C(t)}\right) N(t).$$

The equation has the usual term $\eta L_R(t)$ that represents the growth in varieties due to R&D in the private sector. The new term in the equation is $\phi \frac{g(t)}{C(t)}$, where $\phi > 0$ is a parameter and $\frac{g(t)}{C(t)}$ is the ratio of government spending to output (consumption index $C(t)$ is the measure of output in this model). The term $\phi \frac{g(t)}{C(t)}$ represents the growth in varieties due to R&D by the government.

The government balances its budget every period, so $g(t)$ must be equal to the tax revenue collected. Firm profit after taxation is given by:

$$\pi(t) = (p(t)y(t) - w(t)l(t))(1 - \tau).$$

There are a total of $N(t)$ firms, meaning that total tax revenue is equal to $\tau (p(t)y(t) - w(t)l(t)) N(t)$.

- (a) Write down the expression for firm profit and the free entry condition for firms (Hint: the free entry condition is the same as in the notes).
- (b) Consider a balanced growth path in which the research labor supply is L_R , the growth rate for consumption is g_C , the growth rate for varieties is g_N , and the growth rate for profit is g_π . Write down Equations 1, 2, 3, and 4 that characterize a solution for the balanced growth path (Hint: Equations 2 and 3 are the same as in the notes).
- (c) Solve for the balanced growth path research labor supply L_R .
- (d) How big does the value ϕ need to be so that the growth rate for varieties g_N is higher with both private sector and government compared to private sector alone? Does ϕ need to be bigger than η ? Provide either economic intuition or a mathematical argument (the latter is preferred). Recall that if the growth rate for varieties g_N is higher, then the growth rate for consumption g_C is higher as well.

3. Fiscal Policy (25 points)

Consider the neoclassical growth model with households, firms, and a government. Households supply labor inelastically ($n_t = 1$) and choose consumption c_t , capital stock k_t , and investment i_t to maximize utility subject to a budget constraint and a law of motion for capital. The utility function is given by:

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t),$$

where β is the discount rate parameter. The budget constraint is given by:

$$c_t + i_t(1 + \tau) \leq R_t k_t + w_t + T_t,$$

where R_t is the rate of return on capital, w_t is the wage rate, τ is the investment tax rate set by the government, and T_t are the lump-sum transfers received from the government. The capital and labor markets are competitive, so households take R_t and w_t as given. The law of motion for the capital stock is given by:

$$k_{t+1} = (1 - \delta) k_t + i_t,$$

where δ is the depreciation rate parameter. Firms choose capital input K_t and labor input N_t to maximize profit given their available production function:

$$f(K_t, N_t) = K_t^\theta N_t^{1-\theta},$$

where θ is the production parameter. The government chooses the tax rate τ and receives tax revenue τi_t in each period. The government must balance its budget each period, meaning that transfers to households must be equal to tax revenue: $T_t = \tau i_t$.

- (a) Write down the equations for the steady state of the model as a function of the parameters $(\beta, \delta, \theta, \tau)$.
- (b) Suppose the government optimally chooses the investment tax rate τ^* to maximize tax revenue τi_t . Derive (meaning show the necessary steps) a formula for the revenue-maximizing tax rate τ^* as a function of the remaining parameters (β, δ, θ) .
- (c) Find the value for the revenue-maximizing tax rate τ^* when $(\beta, \delta, \theta) = (0.95, 0.125, 0.35)$.

4. Leverage Cycle (25 points)

Consider a 2-period leverage cycle model with one asset and one commodity. In the second period, two possible states can occur (Good and Bad). The payout of the asset in the Good state is 1 and the payout of the asset in the Bad state is $d(B) = 0.4$. The initial endowment of the commodity equals $E = 2$ for all households. The initial endowment of the asset equals 1 for all households. There exists a unit mass of households in the economy. The household beliefs h are uniformly distributed in the set $[Lo, Hi] = [0.25, 0.75]$, where household h believes the Good state occurs with probability h and the Bad state occurs with probability $1 - h$.

Allow the households to borrow, where borrowing is secured with collateral.

- (a) Solve for the equilibrium price of the asset.
- (b) Solve for the equilibrium leverage in the economy.