Final Exam

1. **Investment Taxation in the Neoclassical Growth Model (25 points)**

   Consider a neoclassical growth model in which the government raises revenue only through investment taxation, households supply labor inelastically, and the firm production function is given by $f(K_t, N_t) = K_t^\theta N_t^{1-\theta}$.

   (a) Suppose the government only seeks to maximize the tax revenue. The objective function for the household is $\sum_{t=0}^{\infty} \beta^t \ln(c_t)$, the household budget constraint is

   $$c_t + i_t (1 + \tau) \le R_t k_t + w_t + T_t,$$

   the law of motion for capital is $k_{t+1} = (1 - \delta) k_t + i_t$, and the government budget constraint is $T_t = \tau i_t$. Here, T_t is the tax revenue.

      i. Derive the equation for the steady state output y_{ss}.

      ii. If the parameter values are given by $\theta = 0.35$, $\beta = 0.95$, and $\delta = 0.12$, what is the value for the revenue-maximizing tax rate τ^*?

      iii. At the revenue-maximizing tax rate τ^* and for the parameter values $\theta = 0.35$, $\beta = 0.95$, and $\delta = 0.12$, what is the value for the steady state output y_{ss}?

   (b) Now suppose the government is able to use the tax revenue to produce a public good and seeks to maximize household welfare. The objective function for the household and the government is $\sum_{t=0}^{\infty} \beta^t \{\ln(c_t) + \alpha \ln(g_t)\}$, the household budget constraint is

   $$c_t + i_t (1 + \tau) \le R_t k_t + w_t,$$

   and the government uses the tax revenue τi_t to produce public good $g_t = \tau i_t$.

      i. If the parameter values are given by $\theta = 0.35$, $\beta = 0.95$, $\delta = 0.12$, and $\alpha = 0.5$, what is the value for the utility-maximizing tax rate τ^{**}.

      ii. At the utility-maximizing tax rate τ^{**} and for the parameter values $\theta = 0.35$, $\beta = 0.95$, $\delta = 0.12$, and $\alpha = 0.5$, what is the value for the steady state output y_{ss}? Compare your findings to part (a.iii).

2. New Keynesian Model with a new Taylor rule (25 points)

Now suppose the Taylor rule responds to both inflation and output gap, as follows:

$$i(t) = \delta + \phi_\pi \pi(t) + \phi_y \tilde{y}(t) + \nu(t).$$

The Taylor principle for this version of the Taylor rule requires $\kappa \left(\phi_\pi - 1 \right) + \phi_y \left(1 - \beta \right) > 0$. The monetary policy shock $\nu(t)$ is an $AR(1)$ process

$$\nu(t) = \rho_\nu \nu(t-1) + \epsilon_\nu(t),$$

where $\rho_\nu \in (0,1)$ and $\epsilon_\nu(t)$ is a random shock drawn from a zero-mean normal distribution. For a positive monetary policy shock (an increase in $\nu(t)$), will the inflation rate $\pi(t)$ increase or decrease? Will the output gap $\tilde{y}(t)$ increase or decrease? Justify your responses.

3. **Leverage Cycle (25 points)**

   Consider the extension of the 3-period leverage cycle model with $h \tilde{\ } Unif[0.4, 0.95]$, dividend values

   $$(d(GG), d(GB), d(BG), d(BB)) = (1, 1, 1, 0.3),$$

   and endowment $E = 0.5$.

   (a) Solve for the equilibrium leverage in state 0 and state B.

   (b) For this economy, what is the size of the Bad News Fraction, the Bankruptcy Fraction, and the Deleveraging Fraction?

4. Growth with Technological Change (25 points)

A government is added to the model that taxes firm profit at the rate τ and uses government spending $g(t)$ to increase the size of the research sector according to the following equation:

$$N(t+1) = \left(1 + \eta\left(L_R(t) + L_R^g(t)\right)\right) N(t).$$

As before, $L_R(t)$ is the labor research provided by R&D firms in the markets. The new term in the equation is $L_R^g(t)$, which is the labor research hired by the government.

Firm profit after taxation is given by:

$$\pi(t) = \left(p(t)y(t) - w(t)l(t)\right)(1 - \tau).$$

With $N(t)$ firms, the tax revenue collected equals

$$\tau\left(p(t)y(t) - w(t)l(t)\right)N(t).$$

This revenue is used to pay the wages of the government research sector. The government balances its budget every period, meaning:

$$w(t)L_R^g(t) = \tau\left(p(t)y(t) - w(t)l(t)\right)N(t).$$

The labor market clearing condition is updated as follows:

$$L_R(t) + L_R^g(t) + l(t)N(t) = 1.$$

Write down the expression for firm profit and the free entry condition for firms (Hint: the free entry condition is the same as in the notes).

Consider a balanced growth path in which both the private sector research labor supply $L_R(t)$ and the government research labor supply $L_R^g(t)$ are constant. Along the balanced growth path, the unknown variables are the research labor supply L_R, the growth rate for consumption g_C, the growth rate for varieties g_N, and the growth rate for profit g_π. Write down Equations 1, 2, 3, and 4 that characterize a solution for the balanced growth path (Hint: Equations 2 and 3 are the same as in the notes, but Equations 1 and 4 need to be updated).

Solve for the balanced growth path research labor supply L_R. The government policy seeks to maximize the growth rate for varieties g_N. Recall that if the growth rate for varieties g_N is higher, then the growth rate for consumption g_C is higher as well. For what parameter values does an increase in the tax rate τ lead to an increase in the growth rate for varieties g_N?