

Final Exam
Tuesday, May 6

1. Topic 1: Macroeconomic Accounting (20 points)

Consider the following table for a 2-commodity economy:

	Quantity	Price
	2012	
Commodity 1	4	\$2
Commodity 2	3	\$3
	2013	
Commodity 1	5	\$2.50
Commodity 2	5	\$3

- (a) Compute the nominal GDP in 2012.
- (b) Assuming 2012 is the base year, compute the real GDP in 2013 (specified in terms of 2012 prices).
- (c) Assuming 2012 is the base year (with $CPI = 100$), compute the CPI in 2013.

2. Topic 2: Microfoundations (20 points)

Solve for the Arrow-Debreu equilibrium (price and allocation) for the following economy with two households and two commodities (hint: here, the expected utility weights are given by $\alpha_1^1 = \alpha_2^1 = \alpha_1^2 = \alpha_2^2 = \frac{1}{2}$):

$$U^1(x_1^1, x_2^1) = \frac{1}{2}\sqrt{x_1^1} + \frac{1}{2}\sqrt{x_2^1},$$

$$U^2(x_1^2, x_2^2) = \frac{1}{2}\sqrt{x_1^2} + \frac{1}{2}\sqrt{x_2^2},$$

$$(e_1^1, e_2^1) = (1, 8),$$

$$(e_1^2, e_2^2) = (3, 1).$$

3. Topic 3: Growth (20 points)

Consider an infinite time horizon problem with a household and a firm. The household takes as given the sequence of prices $\{R_t, w_t\}$ and chooses how much to consume, invest, and work $\{c_t, k_t, n_t\}$. The objective function of the household is given by the infinite discounted utility:

$$\sum_{t=0}^{\infty} \beta^t \{\ln(c_t) - \gamma n_t\}.$$

The budget constraint of the household is

$$c_t + k_{t+1} \leq R_t k_t + w_t n_t.$$

The firm maximizes its profit in every time period. The firm takes as given the sequence of prices $\{R_t, w_t\}$ and chooses the production inputs $\{K_t, N_t\}$. The production function for the firm is given by

$$f(K_t, N_t) = (K_t)^\theta (N_t)^{1-\theta}.$$

- (a) Write down the Bellman equation for the household's optimization problem.
- (b) As a function of the parameters (β, θ, γ) , write down expressions for the policy function $g(k)$, the consumption function $c(k)$, and the labor supply function $n(k)$.

4. Topic 4: Classical Monetary Theory (20 points)

Consider the monetary search model with bargaining. A fraction $M \in (0, 1)$ of all agents are buyers that are currently endowed with one unit each of the fiat currency. The remaining fraction $1 - M$ of all agents are sellers with the ability to produce a perishable consumption good. Agents have utility over the consumption good given by the utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ such that $u(0) = 0$ and u is strictly increasing and strictly concave. Each agent incurs a linear production cost so that producing Q units results in a utility cost of Q . The discount factor is β .

Assume that "double coincidence of wants" is not possible. That is, if two sellers are randomly paired, then they are unable to trade (hint: the parameter γ from the notes is the probability of "double coincidence of wants" and the only difference between the notes and this question is that $\gamma = 0$ in this question).

The value function for the buyer is

$$V_b = \beta(1 - M) [u(q) + V_s] + \beta M V_b.$$

The value function for the seller is

$$V_s = \beta(1 - M)V_s + \beta M [V_b - Q].$$

Define the difference $D(Q) = V_b - V_s$. In equilibrium, $q = \max \{D(Q), 0\} = Q$.

- (a) Plot Q on the x-axis and $\max \{D(Q), 0\}$ on the y-axis. An equilibrium is any intersection of $\max \{D(Q), 0\}$ with the 45-degree line.
- (b) How many nonmonetary equilibria are there?
- (c) How many monetary equilibria are there?

5. Topic 5: Lagniappe (20 points)

Consider a 2-period leverage cycle model with one asset and one commodity. In the second period, two possible states can occur (Good and Bad). The payout of the asset in the Good state is 1 and the payout of the asset in the Bad state is 0.5. The initial endowment of the commodity equals 1 for all households. The initial endowment of the asset equals 1 for all households. As in the notes, there are a continuum of households $h \in [0, 1]$, where household h has the probability h that the Good state will occur (and probability $1 - h$ that the Bad state will occur).

Allow the households to borrow, where borrowing is secured with collateral.

- (a) What borrowing contracts are traded in equilibrium (how much can be borrowed and how much collateral is required)?
- (b) Solve for the equilibrium price of the asset in the initial period.
- (c) What fraction of households are borrowing in this economy?