1. **Neoclassical growth model with home production**

   Consider a model with home production. Household consumption of the market commodity is denoted c_M and household consumption of the commodity produced at home is denoted c_H.

   Assume that there is a set of homogeneous consumers in an economy with an infinite time horizon. The preferences are

$$\sum_{t=0}^{\infty} \beta^t u\left(c_{M,t}, c_{H,t}\right),$$

   where $c_{M,t}$ is the level of market consumption at time t and $c_{H,t}$ is the level of home consumption at time t. Each consumer has a unit of time in each period that can be split between labor on the market and labor for home production. Denote n_t as the amount of labor supplied on the market.

   The utility function is of the constant elasticity of substitution (CES) form:

$$u\left(c_{M,t}, c_{H,t}\right) = \left(\theta\left(c_{M,t}\right)^{\epsilon} + (1-\theta)\left(c_{H,t}\right)^{\epsilon}\right)^{1/\epsilon}.$$

   The elasticity is always constant and equal to $\frac{1}{1-\epsilon}$. The parameter $\epsilon < 1$. As $\epsilon \to -\infty$, the commodities become perfect complements (elasticity equals 0), while as $\epsilon \to 1$, the commodities become perfect substitutes (elasticity equals ∞).

   The budget constraint for the consumers is standard:

$$c_{M,t} + k_{t+1} \leq R_t k_t + w_t n_t.$$

   Home production occurs using the following home production function:

$$c_{H,t} = A\left(1 - n_t\right).$$

   A is the productivity parameter.

   There is a single firm in the economy with production function of the standard Cobb-Douglas form:

$$f\left(K_t, N_t\right) = \left(K_t\right)^{\theta}\left(N_t\right)^{1-\theta} \text{ for } \theta \in (0,1),$$

   where K_t is the total capital stock in period t, and N_t is the total labor supply in period t.

   (a) State the Contraction Mapping Theorem.

   (b) The mapping $T : CB(A) \to CB(A)$ is defined as follows:

$$\left(TV_n\right)(k) = \max_{\left(c_M, c_H, k', n\right) \in \Gamma(k)} u\left(c_M, c_H\right) + \beta V_n\left(k'\right)$$

   for any value function $V_n : A \to \mathbb{R}$ and any level of capital stock $k \in A$.
   Define the budget correspondence Γ.

   (c) Fill in the blanks with the assumptions needed for Berge's Theorem:
   If $u\left(c_M, c_H\right) + \beta V_n\left(k'\right)$ is continuous and Γ is _________, then $TV_n : A \to \mathbb{R}$ is continuous.

   (d) What is Blackwell's 1st sufficiency condition? Show that T satisfies this condition.

(e) Define a recursive competitive equilibrium (RCE). Make sure that you clearly distinguish between aggregate state variables and individual state variables.

(f) Suppose in the current period that the productivity for home production suddenly increases from $\underline{A}$ to $\overline{A}$ and stays at this new higher level forever. What effect does this have on the market labor supply, market output, return on capital, and wage rate?

To answer this question correctly, you must write down the equilibrium equations (i.e., the first order conditions and Euler equations).