ECON512 Intermediate Economics II: Macroeconomics **Matthew Hoelle**
Spring 2015 **Purdue University**

Final Exam
Thursday, May 7, 3:30-5:30 pm, LYNN G167

1. **Growth Theory (40 points)**

   Consider an infinite time horizon problem with a household and a firm. The household takes as given the sequence of prices $\{R_t, w_t\}$ and chooses how much to consume, invest, and work $\{c_t, k_t, n_t\}$. The objective function of the household is given by the infinite discounted utility:

   $$\sum_{t=0}^{\infty} \beta^t \{\ln(c_t) - \gamma n_t\}.$$

   Suppose that investment is taxed at the rate τ. The budget constraint of the household is

   $$c_t + (1+\tau) k_{t+1} \leq R_t k_t + w_t n_t.$$

   The firm maximizes its profit in every time period. The firm takes as given the sequence of prices $\{R_t, w_t\}$ and chooses the production inputs $\{K_t, N_t\}$. The production function for the firm is given by

   $$f(K_t, N_t) = (K_t)^{\theta} (N_t)^{1-\theta}.$$

   (a) Write down the household Euler equation for this problem.

   (b) As a function of the parameters $(\beta, \theta, \gamma, \tau)$, solve for the policy function $g(k)$.

   (c) Consider a 2-country model with identical parameters $(\beta, \theta, \gamma) = (0.95, 0.35, 3.5)$ in both countries. Suppose that $\tau_1 = 0.4$ and $\tau_2 = 0.1$ (the values of the investment tax in each country). What is the ratio of country 1 steady state output relative to country 2 steady state output?

2. **New Keynesian Model (30 points)**

In the New Keynesian model, consider the effects of a technology shock (set the monetary policy shock $\nu(t) = 0$). The technology shock is an $AR(1)$ process

$$a(t) = \rho_a a(t-1) + \epsilon_a(t),$$

for $\rho_a \in (0,1)$ and where $\epsilon_a(t)$ is a random shock drawn from a normal distribution $N\left(0, \sigma_a^2\right)$. The optimal solution to the system of equations

$$\begin{pmatrix} \tilde{y}(t) \\ \pi(t) \end{pmatrix} = A \begin{pmatrix} E_t\{\tilde{y}(t+1)\} \\ E_t\{\pi(t+1)\} \end{pmatrix} + B\left(\sigma \psi_{ya}^n E_t\{a(t+1) - a(t)\}\right)$$

is such that

$$\tilde{y}(t) = \left(-\frac{\sigma \psi_{ya}^n (1-\rho_a)(1-\rho_a \beta)}{\sigma(1-\rho_a)(1-\rho_a\beta) + \kappa(\phi_\pi - \rho_a)}\right) a(t).$$

Recall that the output gap is defined as:

$$\tilde{y}(t) = y(t) - y^n(t)$$

and the natural output level is defined in terms of the parameters $\left(\psi_{ya}^n, \xi_y^n\right)$:

$$y^n(t) = \psi_{ya}^n a(t) + \xi_y^n.$$

(a) Write down an expression for $y(t)$ as a function of $a(t)$.

(b) If $a(t)$ increases, will the output $y(t)$ increase, decrease, or stay the same? Justify your response. For this question, you may use the fact that $0 < \psi_{ya}^n < 1$.

3. **Leverage Cycle (30 points)**

Consider a 2-period leverage cycle model with one asset and one commodity. In the second period, two possible states can occur (Good and Bad). The payout of the asset in the Good state is 1 and the payout of the asset in the Bad state is 0.5. The initial endowment of the commodity equals 1 for all households. The initial endowment of the asset equals 1 for all households. There exists a unit mass of households in the economy. Different from the notes, the households are uniformly distributed in the set $[0.25, 0.75]$, where household h has the probability h that the Good state will occur (and probability $1-h$ that the Bad state will occur).

Allow the households to borrow, where borrowing is secured with collateral.

(a) What fraction of households are borrowers in the economy?

(b) Solve for the equilibrium price of the asset in the initial period.

(c) Solve for the equilibrium leverage in the economy.