Final Exam
Monday, October 19, 2015

Directions

- You may use a calculator for this exam. You are not permitted to access user-generated files in this calculator.
- This is an 90 minute exam worth 100 points.
- Justify your answers by showing work.
- If you feel that additional assumptions are required to answer the question, state these assumptions clearly.

1. **Neoclassical growth model with human capital accumulation**

   Consider a growth model with human capital. An economy consists of a continuum of identical, infinitely lived households. Households have the standard CRRA utility function with coefficient of risk aversion σ. The lifetime utility is $\sum_{t=0}^{\infty} \beta^t \frac{(c_t)^{1-\sigma}}{1-\sigma}$. Households are endowed with one unit of time in each period and choose to spend n_t amount of time working and $1 - n_t$ increasing the human capital h_t. The law of motion for human capital is given by:

   $$h_{t+1} = \Omega h_t (1 - n_t)$$

   for some parameter $\Omega > 1$.

   The model does not include any taxation and physical capital is fully depreciated every period. Labor efficiency is defined as the product of human capital and labor supply. Households receive a wage rate for each unit of labor efficiency provided, meaning that total labor income is a function of both human capital and labor supply. The rate of return on physical capital is given by R_t and the wage rate (for each unit of labor efficiency) is given by w_t.

   In additional to the human capital law of motion, the households face the budget constraint:

   $$c_t + k_{t+1} \leq R_t k_t + w_t (h_t n_t).$$

   The firm production function is given by:

   $$Y_t = (K_t)^{\alpha} (H_t N_t)^{1-\alpha}.$$

   In equilibrium, markets for physical capital, human capital, and labor all must clear, meaning that $K_t = k_t$, $H_t = h_t$, and $N_t = n_t$.

   (a) State the Contraction Mapping Theorem.

   (b) State Berge's Maximum Theorem.

   (c) Define the appropriate mapping $T : CB(A) \to CB(A)$ and the appropriate budget correspondence Γ required for Berge's Maximum Theorem and the Contraction Mapping Theorem.

   (d) Verify that the budget correspondence Γ is both upper and lower hemicontinuous.

   (e) Define a recursive competitive equilibrium (RCE). Make sure that you clearly distinguish between aggregate state variables and individual state variables.

   (f) A balanced growth path is defined such that consumption, physical capital, and human capital grow at a constant rate and the labor supply n_t is a constant. Denote the growth rates as:

   $$1 + \gamma_c = \frac{c_{t+1}}{c_t} \quad 1 + \gamma_k = \frac{k_{t+1}}{k_t} \quad 1 + \gamma_h = \frac{h_{t+1}}{h_t}.$$

   What is the relationship between γ_h and γ_k? What is the relationship between γ_c and γ_k?